Skew Information for a single Cooper Pair Box interacts with a single cavity Mode

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Abstract

The dynamics of the skew information (SI) is investigated for a single Cooper Pair Box,CPB interacts with a single cavity mode. The effect of the cavity and CPB's parameters on the SI is discussed. We show that, it is possible to increase the skew information to reach its maximum value either by increasing the number of photons inside the cavity or considering non-resonant case with larger detuning parameter. The effect of the relative ratio of Josephson junction capacity and the gate capacity is investigated, where the number of oscillations of the skew information increases by decreasing this ratio and consequently the travelling time between the maximum and minimum values decreases.

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1 Introduction

Cooper pairs represent one of the most important promising candidate in the context of quantum information filed. These pairs are classified as a physical realization of the solid states qubit [1]. In quantum teleportation, the generated entangled state between the Cooper pair Box CPB and the cavity mode is used as a quantum channel to perform the original [2] quantum teleportation protocol [3]. In the presence of noise these channels are employed to implement the original quantum teleportation protocol [**?**]. The quantum computational speed of a single Cooper Pair box is investigated by evaluation the speed of orthogonality[14]. Moreover, the generated channel between a single CPB and a cavity mode is used to send coded information in the presence of perfect and imperfect operation during the coding process [6]. The dynamics of the purity, entropy and the coherent vector of a single the Cooper pair interacts with a single cavity mode is investigated in [7].

Recently, skew information has been used as a test for quantum entanglement , where the Bell inequality is obtained by means of the skew information. This new inequality provides an exact test to distinguish entangled from non-entangled pure states of two qubits [8]. Pezzè et.al has introduced a measure of entanglement between multi-particles in terms of Fisher information [9]. Luo [10] has find a mathematical between the Fisher information, which many applications in quantum information and the skew information. This motivates us to investigate the skew information for a single Cooper Pair Box (CPB) interacts with a single cavity mode..

In this contribution we investigate the dynamics of the skew information for a single Cooper pair box interacts with a single cavity mode. The effect of the cavity and the Cooper pair box's parameters on the dynamics of the skew information is discussed. We show that the skew information increases faster for larger values of photons inside the cavity or increasing the detuning parameter between the cavity and the CPB. Also, the effect of the relative ratio of Josephson junction capacity and the gate capacity on the behavior of the skew information is investigated

The paper is organized as follows: in Sec. 2, we introduce a brief discussion on the qubit cavity interaction and its dynamics. Sec 3 is devoted to the skew information, where we review its definition in the context of quantum information and its relation to the degree of entanglement. Also, we discuss the dynamics of the skew information for different values of the CPB and cavaity's parameters. Finally, the results are concluded in Sec.4.

2 The Suggested Model

A single Cooper Pair Box CPB is an example of a qubit with states $|0\rangle$ and $|1\rangle$ [11]. It consists of a small superconducting island connected to the outside by Josephson tunnel junction E_j and a gate capacitor C_J. A gate voltage V_g is coupled to the superconducting island through gate capacitance C_g [12, 13]. This system can be described by two levels system with hamiltonian

$$H_s = 4E_c(n - n_g)^2 - E_j \cos\phi, \tag{1}$$

where, $E_c = \frac{1}{2}e^2\left(C_J + C_g\right)$ is the charging energy, $E_J = \frac{1}{2}\frac{\hbar}{e}I_c$ is the Joesphson coupling energy, e is the charge of the electron, $n_g = \frac{1}{2}\frac{V_g}{e}C_g$ is the dimensionless gate charge, n is the number operators of excess Cooper Pair on the island and ϕ is the phase operator [15, 16]. The Hamiltonian of the system (1) reduces to,

$$H_s = -\frac{1}{2}B_z\sigma_z - \frac{1}{2}B_x\sigma_x, \tag{2}$$

where it is assumed that the temperature is low enough and Josephon coupling energy, E_j is much smaller than the charging energy i.e. $E_j << E_c$ and $B_z = -\left(2n-1\right)E_{cl}$, E_{cl} is the electric energy, $B_x = E_j$ and $\sigma_x, \sigma_y, \sigma_z$ are Pauli matrices [4]. If the single Cooper pair Box, CPB is placed inside a single-mode microwave cavity, then the Hamiltonian of the system can be written as [14],

$$\mathcal{H} = \varpi a^\dagger a + \varpi_c\sigma_z - g\{\mu - \cos\theta\sigma_z + \sqrt{1-\nu^2}\sigma_x(a^\dagger + a)\}, \tag{3}$$

where, ω is the cavity resonance frequency, $\omega_c = \sqrt{E_j^2 + 16E_c^2\left(2n_g - 1\right)^2}$ is the transition frequency of the cooper pair qubit, $g = \frac{\sqrt{C_j}}{C_{g+C_J}}\sqrt{\frac{1}{2}\frac{\varpi}{\hbar}e^2}$ is coupling strength of resonator to the Cooper Pair Box, $\mu = 1 - n_g$, and $\theta = -\arctan\left(\frac{1}{E_c}\frac{E_j}{2n_g-1}\right)$ is mixing angle.

Let us assume that a single Cooper pair, prepared initially an a superposition state as $|\psi_c(0)\rangle = \frac{1}{\sqrt{2}}(|e\rangle + |g\rangle)$ interacts with a single cavity mode, prepared initially in a number state, $|\psi_f\rangle = |n\rangle$. In this case, the initial state of the total system is defined by $|\psi_s(0)\rangle = \frac{1}{\sqrt{2}}(|e,n\rangle + |g,n\rangle)$. The time evolution of the initial state vector is given by,

$$|\psi_s(t)\rangle = \mathcal{U}(t)\,|\psi_s(o)\rangle\,, \tag{4}$$

where, $\mathcal{U}(t)$ is a unitary operator defined by,

$$\mathcal{U}(t) = \mathcal{B}_1|e\rangle\langle e| + \mathcal{B}_2|e\rangle\langle g| + \mathcal{B}_3|g\rangle\langle e| + \mathcal{B}_4|g\rangle\langle g|, \tag{5}$$

where,

$$
\begin{aligned}
\mathcal{B}_1 &= C_{n+1} - i\delta S_{n+1},\ \mathcal{B}_2 = -iS_n a,\\
\mathcal{B}_3 &= iS_n a^{\dagger},\ \mathcal{B}_4 = c_{n+1} + i\delta S_{n+1},\\
C_n &= \cos\Omega\tau\sqrt{(\Delta^2+n}, \quad S_n = \frac{2\lambda}{\sqrt{\Delta^2+4g^2 n}}\sin\Omega\tau\sqrt{\Delta^2+n},\\
\Omega &= \frac{\sqrt{c_j}}{C_j+C_g}, \quad T = \sqrt{\frac{\omega}{2\hbar}} \text{ and } \Delta = \frac{\delta}{2g}.
\end{aligned} \tag{6}
$$

Using Eq.(5), then the state vector (4) becomes,

$$
|\psi_s(t)\rangle = \mathcal{A}_1|e,n\rangle + {}_2|g,n+1\rangle + \mathcal{A}_3|e,n-1\rangle + \mathcal{A}_4|g,n\rangle, \tag{7}
$$

where

$$
\mathcal{A}_1 = \frac{\mathcal{B}_1}{\sqrt{2}},\ \mathcal{A}_2 = \frac{\mathcal{B}_1}{\sqrt{2}}\sqrt{n+1},\ \mathcal{A}_3 = \frac{\mathcal{B}_1}{\sqrt{2}}\sqrt{n},\ \mathcal{A}_4 = \frac{\mathcal{B}_4}{\sqrt{2}}. \tag{8}
$$

Since, the final state of the initial state vector is obtained, then we can investigate all the classical and quantum phenomena associated with this quantum state vector. In this context, we are interested to investigate the dynamics of the Skew information.

3 Skew Information

Skew information ($\mathcal{S}_I$) represents a measure in information content of the density operator with respect to a self adjoint operator [17]. Mathematically for a density operator ϱ and a self- adjoint operator $\mathcal{H}$, the skew information is defined by

$$
\mathcal{S}_I = \frac{1}{2}tr\{\sqrt{\varrho}\mathcal{H} - \mathcal{H}\sqrt{\varrho}\}^2. \tag{9}
$$

Therefore one can say that $\mathcal{S}_I$ measures the non-commutativity between ϱ and $\mathcal{H}$ [10]. Moreover, it has been shown that the skew information can by used to detect entanglement, where the Bell inequality is proposed in terms of $\mathcal{S}_I$ and , it is proved that the inequality provides an exact test to distinguish entangled from separable pure states of two qubits [8]. In an equivalent form of (9), the skew information can be written as,

$$
\mathcal{S}_I = tr\{\varrho\mathcal{H}^2\} - tr\{\sqrt{\varrho}\mathcal{H}\sqrt{\varrho}\mathcal{H}\}. \tag{10}
$$

To investigate the skew information for the Cooper Pair Box interacts with a cavity mode initially prepared in a number state, we consider a set of spin $\frac{1}{2}$ operators for the CPB and the cavity as σ_i and τ_i , $i = x, y, z$ respectively. Now, the skew information for the CPB and the cavity is defined as

$$
\mathcal{S}_I = \sum_i \{\Delta\sigma_i^2 + \Delta\tau_i^2\}, \tag{11}
$$

where $i = x, y, z$ and $\Delta A^2 = \langle A^2\rangle - \langle A\rangle^2$. Using (7) and (11) one obtains the skew information for the suggested system. However, for a two -qubit pure system the skew information and the concurrence are connected by [18]

$$
\mathcal{S}_I = 1 + \mathcal{C}^2, \tag{12}
$$

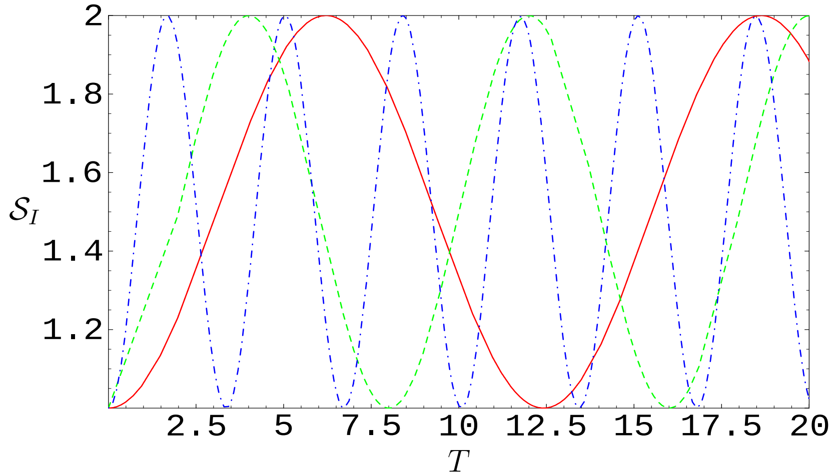

Figure 1: The skew information $\mathcal{S}_I$ between the Cooper pair and the cavity mode for a system initially prepared in $|n,e\rangle$. The solid , dot and dash dot curves are evaluated for $\Delta = 0.0, 0.3$ and 0.9 respectively. The ratio $\gamma = \frac{C_j}{C_g} = \frac{1}{4}$ and the number of photon inside the cavity $n = 1$.

where $\mathcal{C}$ is the concurrence which quantify the degree of entanglement between the CPB and the field, where for two qubits, the concurrence is calculated in terms of the eigenvalues $\eta_1, \eta_2, \eta_3, \eta_4$ of the matrix $R = \rho\sigma_y \otimes \sigma_y \rho^* \sigma_y \otimes \sigma_y$. It is given by $\mathcal{C} = max\{0, \eta_1 - \eta_2 - \eta_3 - \eta_4\}$, where $\eta_1 \geq \eta_2 \geq \eta_3 \geq \eta_4$. For maximally entangled states concurrence is 1 while for separable states it is zero [19].

The dynamics of the skew information for different values of the detuning parameter is displayed in Fig.(1), where it is assumed that the other parameters are constant. It is clear that, for the resonant case i.e. $\Delta = 0.$, the skew information $\mathcal{S}_I$ increases as T increases to reach its maximum value for the first time at $T \simeq= 7$. However for larger τ, the skew information decreases smoothly to reach its minimum value for the first time at $\tau \simeq 12.5$. This behavior is repeated as the scaled time increases. For non resonant case, as example for ($\Delta = 0.3$), the behavior of the skew information is similar to that predicted in the non-resonant case. However, $\mathcal{S}_I$ increases faster and reaches its maximum value for the first time at $T \simeq 3$. As the interaction increases the skew information reaches its minimum values faster than that depicted for resonant case.

Fig.(2), displays the effect of different numbers of photons inside the cavity. In this figure, we consider a non-resonant case, i.e $\Delta = 0.0$ and we fix the ratio $\gamma = \frac{C_j}{C_g} = \frac{1}{4}$. The dynamics of the $\mathcal{S}_I$ is similar to that shown in Fig.(1). However the number of oscillations depends on the number of photons inside the cavity. It is clear that, for larger values of n, the skew information reaches its maximum value faster than that evaluated for smaller number of photons inside the cavity and consequently $\mathcal{S}_I$ vanishes earlier for larger values of n. Comparing Fig.(1)(solid curve) and its corresponding one in Fig.(2), we can see that for $n = 1$, the skew information reaches its minimum value at $T = 12.5$, while for $n = 2$ (solid curve in Fig.2), the skew information reaches its minimum value at $T = 10$.

The effect of the ratio γ is displayed in Fig.(3), for resonant and non-resonant cases, where we fix the other parameters. It is clear that for larger values of γ, the skew information reaches its minimum values faster than that shown for smaller values of γ. Also, for smaller values of γ, $\mathcal{S}_I$ increases gradually but for larger γ the skew information increases hastily. This behavior is clearly displayed in Fig.(3a) for resonant case. As one increases the detuning

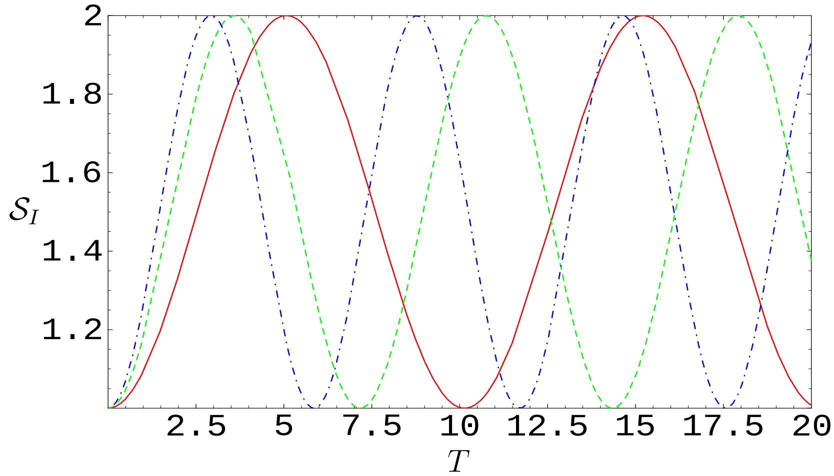

Figure 2: The same as Fig.1, but for different values of the photon inside the cavity. The solid , dot and dash dot curves are evaluated for $n = 2, 5$ and 8 respectively. The ratio $\gamma = \frac{C_j}{C_g} = \frac{1}{4}$ and the detuning parameter $\Delta = 0.0$.

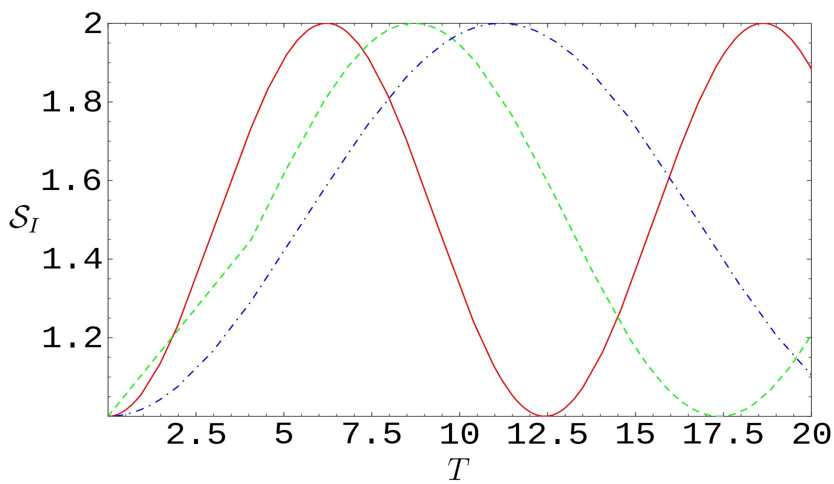

Figure 3: The same as Fig.1, but for different values of the the ratio γ. The solid, dot and dash dot curves are evaluated for $\gamma = \frac{1}{4}, \frac{1}{6}$ and $\frac{1}{8}$ respectively. The number of photons inside the cavity $n = 2$, and the detuning parameter (a) for resonant case i.e .$\Delta = 0.0$ (b) for non-resonant case with $\Delta = 0.3$.

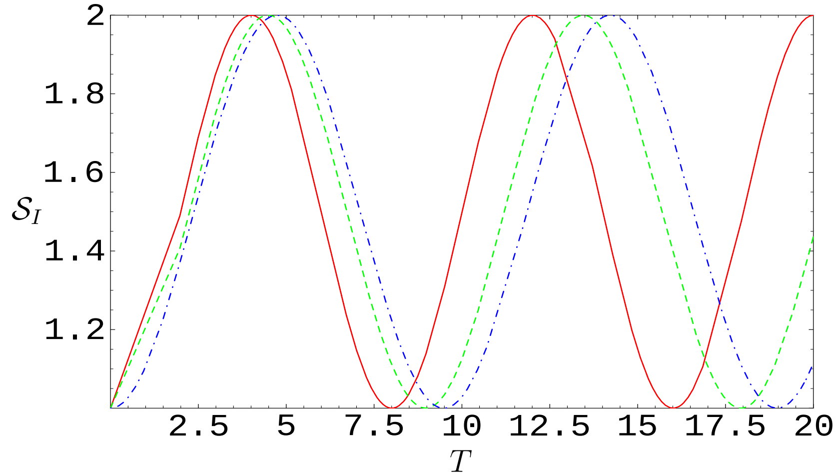

Figure 4: The same as Fig.1, but for different values of the the ratio γ. The solid, dot and dash dot curves are evaluated for $\gamma = \frac{1}{4}, \frac{1}{6}$ and $\frac{1}{8}$ respectively. The number of photons inside the cavity $n = 2$, and the detuning parameter (a) for resonant case i.e .$\Delta = 0.0$ (b) for non-resonant case with $\Delta = 0.3$.

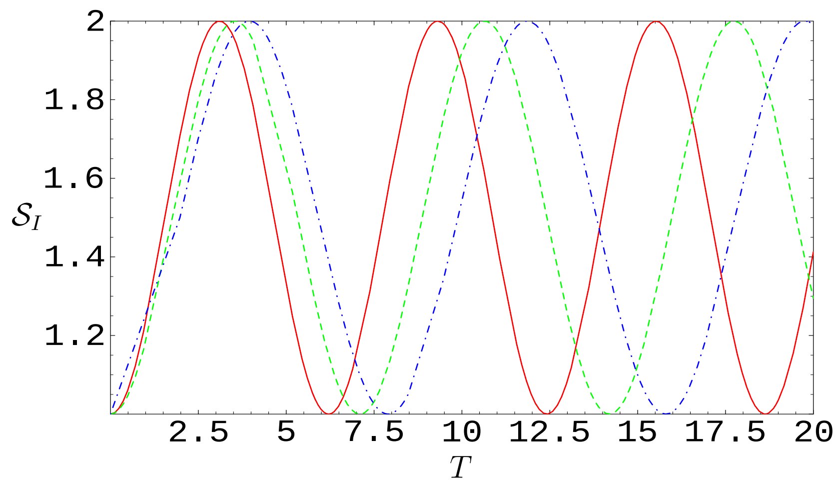

Figure 5: The same as Fig.1, but for different values of the ratio γ. The solid, dot and dash dot curves are evaluated for $\gamma = 4, 6$ and 8 respectively while the and the number of photons inside the cavity $n = 2$, and the detuning parameter $\Delta = 0.0$.

by 30%, the behavior of $\mathcal{S}_I$ changes dramatically. The skew information increases and decreases hastily regardless of the ratio γ. In this case the minimum values of $\mathcal{S}_I$ is reached much earlier i.e T reduces by5%.

Fig.(5) describes the dynamics of $\mathcal{S}_I$ for different values of $\gamma = 2, 4, 6$ and it is assumed that the initial system is prepared in resonant case. It is clear that, the behavior is similar to that shown in Fig.(3b). This show that one can control on the behavior of the skew information either by the detining parameter or the ratio γ.

4 Conclusion

In this contribution we investigate one of the most quantity of information, skew information, for a particle of CPB which is a promising candidate for quantum information and computations. The suggested model consists of a single Cooper pair Box prepared initially in the excited state interacts with a cavity mode prepared in the number state.

The effect of the cavity and the Cooper pair Box's parameters on the dynamics of the skew information is investigated. Different cases are considered the resonant and non-resonant cases. For resonant case, the skew information increases gradually and takes more time to reaches its maximum or minimum values. However for non resonant case, the skew information increases faster for larger values of the detuning parameter and consequently the number of oscillations between the maximum and minimum values increases.

The number of photons inside the cavity has a noticeable effect, where as one increases the number of photons, the skew information increases faster and the oscillation increases. However, if we increase the number of photon by 1%, the oscillations time between the maximum and minimum values reduced by 2%.

The effect of the relative ratio of Josephson junction capacity and the gate capacities plays in important role on the the dynamics of skew information. For small values of this ratio the oscillations of the skew information increases and consequently the revival time decreases. On the other hand the skew information increases faster for smaller values of the ratio of junction capacity and the gate capacities and gradually for larger values of this ratio.

In conclusion: it is possible to control on the dynamics of the skew information by controlling on the cavity or the CPB's parameters. Therefore one can speed up the skew information to reach its maximum value either by increasing the number of photon inside the cavity or considering non-resonant case with larger detuning.

References

[1] Yu. A. Pashkin, T. Yamamoto, O. Astaev, Y. Nakamura, D. Averin, T. Tilma, F. Nori, J. S. Tsai, Physica C426-431 1552 (2005).

[2] C. H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Pweres and W. K. Wootters, Phys. Rev. Lett. **70** 1895 (1993)

[3] N. Metwally and A. A.A. El-Amin, Physica E**41** 718, (2009).

[4] A.-H. M. Ahmed, M. N. Zakaria and N. Metwally,Appl. Math. Inf. Sci.**6** 1 (2012).

[5] A.-S, Obada, N. Metwally, D.M. Abo-kahla and M. abdel-Aty, Physica:E **43**1792 (2011).

[6] N. Metwally, J. Opt. Soc. Am. B **3**- 389 (2012).

[7] A. Ahmed, L. Cheong, N. Zakaria1 and N. Metwally,"Dynamics of Information Coded in a Single Cooper Pair Box" to appear in IJTP. (2012).

[8] Z. Chen, Phys. Rev. A. **71** 052302 (2005).

[9] L. Pezzè and A. Smerzi, Phys. Rev. Lett.**102** 100401 (2009).

[10] S. Luo, Proceding of Am math. Soc. **132** No 3 885-890 (2003).

[11] S. Bose and G. S. Agarwal, New journal of physics B **8** 34 (2006).

[12] Y. Nakamura, Yu. A. Pashkin and J. S. Tsai, Nature **398** 798 (1999).

[13] J. S. Tsai, Y. Nakamura, Physica C 367, 191 (2002).

[14] A.-S. Obada, N. Metwally, D.M. Abo-kahla and M. abdel-Aty,Physica E **43**1792 (2011).

[15] R. Migliore, A. Messina and A. Napoli, Eur. Phys. J. B**13** 585 (2000).

[16] M. Zhang, J. Zou and B. Shao, Int. J. Mod. Phys.**16**4767 (2002).

[17] E. P. Winger and M. M. Yanase, Proc. Nat. Ascd. Sci. USA **49** 910 (1963).

[18] S. Hong-Gui, L. Wan-Fang, L. Chun-Jie, Chin Phys. B**20** 090301 (2011).

[19] W. K. Wootters, Phys. Rev. Lett.**80** 2245 (1998).